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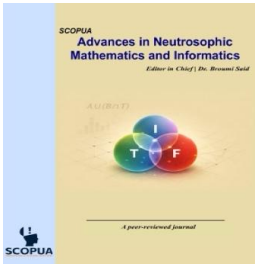

Neutrosophic-Plithogenic Dynamical Systems: Foundations, Stability, Partial and Refined Attractors / Neutrals / Repellers, and Chaos

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ABSTRACT

Classical dynamical systems describe the evolution of states through deterministic or stochastic laws. However, many real-world systems involve uncertainty, contradiction, incompleteness, and indeterminacy that cannot be adequately represented within traditional frameworks. This paper introduces Neutrosophic-Plithogenic Dynamical Systems (NPDS), extending classical dynamics through neutrosophic states and plithogenic contradiction structures. A system state is represented by the neutrosophic triple <T, I, F>, where T, I, and F denote truth, indeterminacy, and falsehood components. Plithogenic attributes and contradiction degrees are then incorporated into the dynamics. New concepts are introduced, including neutrosophic attractors, neutrosophic repellers, neutrosophic neutral sets, neutrosophic partial attractors, neutrosophic partial repellers, neutrosophic Lyapunov exponents, and neutrosophic chaos measures. The resulting framework provides a natural mathematical representation for systems whose behavior is simultaneously attractive, neutral, and repulsive. Applications to scientific theory evolution, social systems, economics, artificial intelligence, and complex decision processes are discussed. The concepts of *Partial Attractors, Partial Neutrals, Partial Repellers* and respectively of *Refined Attractors, Refined Neutrals, Refined Neutral Sets*, were introduced by F.Smarandache in 2026, and they are richer and closer to reality than the ordinary classical versions of Attractors, Repellers.

Keywords: Neutrosophy, Plithogeny, Dynamical Systems, Partial Attractors, Partial Neutrals, Partial Repellers, Refined Attractors, Refined Neutrals, Refined Repellers, Stability, Chaos, Lyapunov Exponents.

1. Introduction

Dynamical systems constitute one of the most important mathematical frameworks for describing evolution over time. Classical models typically represent states as points in a Euclidean space and classify long-term behavior through fixed points, attractors, repellers, bifurcations, and chaos.

Many real-world systems, however, contain uncertainty and contradiction. Scientific theories may be partially supported, partially rejected, and partially unresolved. Social movements may simultaneously attract, repel, and leave neutral different groups. Similar phenomena appear in economics, artificial intelligence, medicine, and cognitive science.

Neutrosophy provides a natural framework for representing such situations through the independent components Truth (T), Indeterminacy (I), and Falsehood (F). Plithogeny further extends this representation by incorporating contradiction degrees among attribute values.

The purpose of this paper is to establish the foundations of Neutrosophic-Plithogenic Dynamical Systems and introduce a new attractor-repeller theory inspired directly by reality.

2. Dynamic Systems

A **dynamical system** is a system whose state changes over time according to a rule or law.

There are **many types of dynamical systems**, classified in different ways.

3. Continuous Dynamical Systems

Time varies continuously, $t \in \mathbb{R}$.

Examples:

- Population growth:

$$\frac{dx}{dt} = rx$$

- Planetary motion
- Electrical circuits
- Fluid dynamics

These are usually described by **differential equations**.

4. Discrete Dynamical Systems

Time advances in steps, $n = 0,1,2, \ldots$.

Examples:

- Economic models
- Iterative algorithms
- Population models with generations

Example:

$$x_{n+1} = f(x_n)$$

The famous logistic map:

$$x_{n+1} = rx_n(1 - x_n)$$

5. Deterministic Dynamical Systems

The future state is completely determined by the current state.

Example:

$$\frac{dx}{dt} = x^2 - 1$$

No randomness is involved.

6. Stochastic Dynamical Systems

Randomness affects the evolution.

Example:

$$dx = (ax + b)\, dt + \sigma\, dW_t$$

where $W_t$is Brownian motion.

Applications:

- Finance
- Biology
- Climate science

7. Linear Dynamical Systems

The governing equations are linear.

Example:

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}$$

Properties:

- Easier to solve.
- Superposition principle holds.

8. Nonlinear Dynamical Systems

The equations contain nonlinear terms.

Example:

$$\frac{dx}{dt} = x(1 - x)$$

Applications:

- Ecology
- Neural networks
- Physics

Nonlinear systems can exhibit chaos.

9. Autonomous Dynamical Systems

The rule does not explicitly depend on time.

$$\frac{dx}{dt} = f(x)$$

Example:

$$\frac{dx}{dt} = x - x^3$$

10. Non-Autonomous Dynamical Systems

The rule explicitly depends on time.

$$\frac{dx}{dt} = f(x,t)$$

Example:

$$\frac{dx}{dt} = x + \sin\ t$$

11. Conservative Dynamical Systems

Energy is conserved.

Examples:

- Ideal planetary motion
- Ideal pendulum

Hamiltonian systems belong here.

12. Dissipative Dynamical Systems

Energy decreases over time.

Example:

- Damped pendulum

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

Such systems often approach an attractor.

13. Chaotic Dynamical Systems

Very sensitive to initial conditions.

Examples:

- Lorenz system
- Double pendulum

The famous Lorenz system:

$$\begin{aligned} \dot{x} &= \sigma(y - x), \\ \dot{y} &= x(\rho - z) - y, \\ \dot{z} &= xy - \beta z. \end{aligned}$$

A tiny change in initial conditions may lead to drastically different trajectories.

14. Hamiltonian Systems

Described by a Hamiltonian H:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Applications:

- Classical mechanics
- Quantum mechanics

15. Hybrid Dynamical Systems

Combine continuous and discrete evolution.

Examples:

- Robots
- Traffic lights
- Digital control systems

16. Network Dynamical Systems

Dynamics occur on a network or graph.

Examples:

- Social networks
- Neural networks
- Power grids

17. Fractional Dynamical Systems

Use derivatives of fractional order.

Example:

$$D^{\alpha}x(t) = f(x(t)), 0 < \alpha < 1$$

Useful for systems with memory effects.

18. Neutrosophic Dynamical Systems

In the neutrosophic framework, the state may be represented by

$$(T, I, F),$$

where:

- T= degree of truth/state realization,
- I= degree of indeterminacy,
- F= degree of falsity/opposite realization.

A neutrosophic dynamical system may evolve as

$$(T_{n+1}, I_{n+1}, F_{n+1}) = \Phi(T_n, I_n, F_n).$$

Applications include:

- Contradictory information systems,
- Decision making,
- Social systems,
- Quantum uncertainty models,
- Neutrosophic attractors, repellers, and limit cycles.

19. Classification of Dynamic Systems by Long-Term Behavior

A dynamical system may contain:

- Fixed points (equilibria)
- Periodic orbits (cycles)
- Quasi-periodic orbits
- Attractors
- Repellers
- Strange attractors
- Chaotic regions

Examples of attractors include the famous Lorenz Attractor and limit cycles such as those arising in the Van der Pol Oscillator.

A broad hierarchy is:

Dynamical Systems → Continuous / Discrete → Deterministic / Stochastic → Linear / Nonlinear → Autonomous / Non-autonomous → Conservative / Dissipative → Chaotic / Non-chaotic → Integer-order / Fractional-order → Classical / Neutrosophic / Plithogenic

The neutrosophic extension is particularly useful when the evolution involves partial truth, partial falsity, and indeterminacy simultaneously.

20. Neutrosophic State Spaces

A neutrosophic state is represented by

$$X = \langle T, I, F \rangle$$

where:

- T = degree of truth,
- I = degree of indeterminacy,
- F = degree of falsehood.

The components are independent.

The neutrosophic state space is

$$N = \{\langle T, I, F \rangle\}.$$

Unlike probability theory,

$$T + I + F$$

is not required to equal 1.

A trajectory is represented by

$$X(t) = \langle T(t), I(t), F(t) \rangle.$$

Thus truth, indeterminacy, and falsehood evolve simultaneously.

21. Neutrosophic Dynamical Systems

A neutrosophic dynamical system is defined by

$$dX/dt = \Phi(X)$$

or

$$X(n+1) = \Phi(X(n))$$

for continuous and discrete systems respectively.

Equivalently,

$$dT/dt = f_T(T,I,F)$$
$$dI/dt = f_I(T,I,F)$$
$$dF/dt = f_F(T,I,F).$$

The indeterminacy component becomes a genuine state variable rather than a secondary statistical quantity.

A simple neutrosophic logistic model may be written as

$$dT/dt = rT(1-T) - \alpha I$$
$$dI/dt = \beta(T+F) - \gamma I$$
$$dF/dt = rF(1-F) - \delta T.$$

This model allows mutual interaction among truth, falsehood, and indeterminacy.

22. Plithogenic Dynamical Systems

Consider attributes

$$v_1, v_2, \ldots, v_n$$

together with contradiction degrees

$$c(v_i, v_j).$$

A plithogenic dynamical state is represented by

$$P = \{(x_i, v_i)\}.$$

The evolution of each attribute depends not only on its own value but also on the contradiction structure among attributes.

This extension is particularly useful for social, economic, and decision-making systems where contradictory factors coexist.

23. Neutrosophic-Plithogenic Dynamical Systems

A Neutrosophic-Plithogenic Dynamical System combines neutrosophic states and plithogenic attributes:

$$NPDS = (N, A, C, \Phi)$$

where:

- N is the neutrosophic state space,
- A is the attribute set,
- C is the contradiction matrix,
- Φ is the evolution operator.

The dynamics depend simultaneously on truth, indeterminacy, falsehood, and contradiction degrees.

24. Neutrosophic Attractors, Neutrals, and Repellers

The most important extension proposed in this paper concerns attractors and repellers.

Classically, a set is either an attractor or a repeller.

In reality, however, a phenomenon is rarely purely attractive or purely repulsive.

Definition

The neutrosophic characterization of an attractor is

$$A = \langle T_A, I_A, F_A \rangle$$

where:

- T_A = attraction degree,
- I_A = neutrality degree,
- F_A = repulsion degree.

Similarly, a repeller is characterized by

$$R = \langle T_R, I_R, F_R \rangle.$$

Classical attractors and repellers become special cases:

$$A = \langle 1,0,0 \rangle$$
$$R = \langle 0,0,1 \rangle.$$

25. Neutrosophic Partial Attractors, Partial Neutral Sets, Partial Repellers

Partial Attractor

A neutrosophic partial attractor satisfies

$$0 < T_A < 1.$$

Example:

$$A = < 0.6, 0.3, 0.2 >.$$

Interpretation:

- 60% attraction,
- 30% neutrality,
- 20% repulsion.

Partial Repeller

A neutrosophic partial repeller satisfies

$$0 < F_A < 1.$$

Example:

$$R = < 0.2, 0.2, 0.7 >.$$

Partial Neutral Set

A neutrosophic neutral set satisfies

$$I_A = \max(T_A, I_A, F_A).$$

Example:

$$N = \langle 0.3, 0.8, 0.2 \rangle.$$

These notions (trichotomy) provide a richer representation of reality than the classical attractor-repeller dichotomy.

26. Stability and Neutrosophic Lyapunov Exponents

Classically, stability is measured through Lyapunov exponents.

We define the neutrosophic Lyapunov vector

$$\Lambda = \langle \lambda_T, \lambda_I, \lambda_F \rangle$$

where:

- λ_T measures truth divergence,
- λ_I measures indeterminacy divergence,
- λ_F measures falsehood divergence.

A system may therefore be stable with respect to truth while remaining unstable with respect to indeterminacy.

This multidimensional stability concept is more appropriate for complex real-world systems.

27. Neutrosophic Chaos and Plithogenic Chaos

Classical chaos is characterized by sensitive dependence on initial conditions.

We define the Neutrosophic Chaos Index

$$NCI = \langle C_T, C_I, C_F \rangle$$

where:

- C_T = truth-chaos,
- C_I = indeterminacy-chaos,
- C_F = falsehood-chaos.

Similarly, contradiction-dependent chaos leads to Plithogenic Chaos.

Different attributes may exhibit different levels of chaotic behavior depending on their contradiction structure.

The concepts of **Refined Neutrosophic Attractors, Refined Neutrosophic Repellers, and Refined Neutrosophic Neutral Sets** are richer and closer to reality than the ordinary neutrosophic versions.

28. Refined Neutrosophic Attractors

In ordinary neutrosophy:

$$A = \langle T_A, I_A, F_A \rangle$$

where:

- T_A= attraction degree,
- I_A= neutrality degree,
- F_A= repulsion degree.

In refined neutrosophy, attraction is decomposed into several types:

$$A = \langle T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s \rangle.$$

Definition

A Refined Neutrosophic Attractor is an invariant set whose attraction is distributed among several attraction categories:

$$RA = \langle T_1, T_2, \dots, T_p; I_1, \dots, I_r; F_1, \dots, F_s \rangle.$$

Example 1: Human Relationships

For a girl evaluating a boy:

$$RA = \langle T_1, T_2, T_3, T_4; I_1; F_1, F_2 \rangle$$

where:

- T_1= physical attractiveness,
- T_2= economic attractiveness,
- T_3= intellectual attractiveness,
- T_4= emotional attractiveness,
- I_1= uncertainty,
- F_1= behavioral repulsion,
- F_2= ideological repulsion.

Example:

$$RA = \langle 0.9,0.3,0.8,0.7; 0.2; 0.1,0.4 \rangle.$$

Interpretation:

- strong physical and intellectual attraction,
- moderate emotional attraction,
- low economic attraction,
- some behavioral repulsion.

29. Refined Neutrosophic Repellers

Definition

A Refined Neutrosophic Repeller is

$$RR = \langle T_1, \dots, T_p; I_1, \dots, I_r; F_1, \dots, F_s \rangle$$

with one or more repulsive components dominating.

Example 2: Technology Rejection

Consider a new technology.

Repulsive factors:

$$F_1 = \text{cost}$$
$$F_2 = \text{complexity}$$
$$F_3 = \text{security concerns}$$

Attractive factors:

$$T_1 = \text{performance}$$
$$T_2 = \text{innovation}$$

Example:

$$RR = \langle 0.6,0.7; 0.2; 0.9,0.8,0.7 \rangle.$$

The technology is attractive but strongly repelled by cost and security concerns.

30. Refined Neutrosophic Neutral Sets

Definition

A Refined Neutrosophic Neutral Set is characterized by dominant refined indeterminacies:

$$RN = \langle T_1, \dots, T_p; I_1, \dots, I_r; F_1, \dots, F_s \rangle.$$

where several I_j are dominant.

Example 3: Scientific Theory

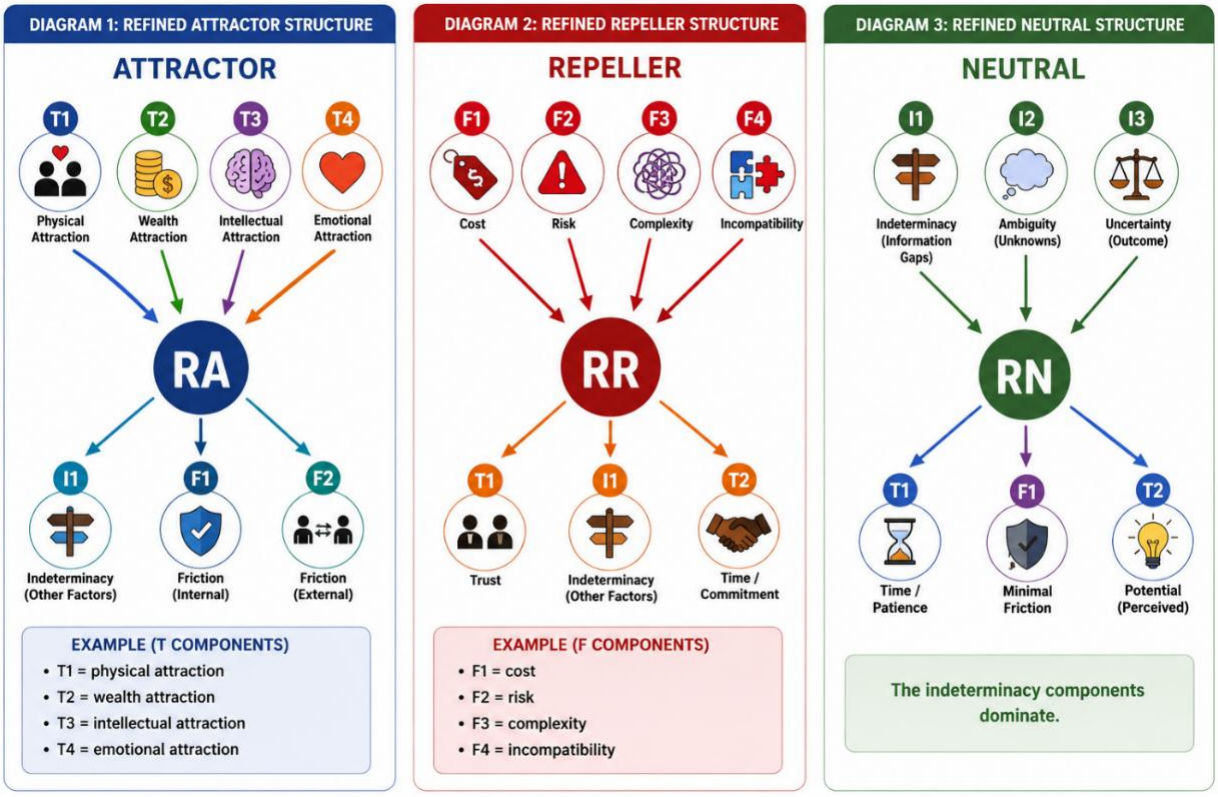
For a scientific theory:

$$I_1 = \text{lack of experiments}$$
$$I_2 = \text{insufficient data}$$
$$I_3 = \text{unresolved mathematics}$$

Example:

$$RN = \langle 0.5, 0.6;\ 0.9, 0.8, 0.7;\ 0.2 \rangle.$$

The theory is neither clearly accepted nor rejected.



Diagrams 1-2-3: Refined Attractor, Refined Repeller, Refined Neutral

Diagram 1: Refined Attractor Structure

Example:

- T_1 = physical attraction
- T_2 = wealth attraction
- T_3 = intellectual attraction
- T_4 = emotional attraction

Diagram 2: Refined Repeller Structure

Example:

- F_1 = cost
- F_2 = risk
- F_3 = complexity
- F_4 = incompatibility

Diagram 3: Refined Neutral Structure

The indeterminacy components dominate.

31. Applications for Refined Attractors, Refined Neutrals, Refined Repellers

Psychology
Refined attraction profiles between people.
Marketing
Consumer attraction:

$$(T_{\text{price}}, T_{\text{quality}}, T_{\text{brand}})$$

Repulsion:

$$(F_{\text{cost}}, F_{\text{risk}})$$

Politics
Attraction:

$$T_{\text{economy}}, T_{\text{security}}$$

Repulsion:

$$F_{\text{corruption}}, F_{\text{taxes}}$$

Neutrality:

$$I_{\text{undecided}}$$

Scientific Theory Evolution
Attractive factors:

$$T_{\text{experiments}}$$
$$T_{\text{predictions}}$$
$$T_{\text{consistency}}$$

Repulsive factors:

$$F_{\text{contradictions}}$$
$$F_{\text{failed tests}}$$

Neutral factors:

$$I_{\text{unknowns}}$$

32. Refined Neutrosophic Attractor Theorem

Every ordinary neutrosophic attractor

$$A = \langle T, I, F \rangle$$

may be refined into

$$RA = \langle T_1, \dots, T_p;\ I_1, \dots, I_r;\ F_1, \dots, F_s \rangle$$

such that

$$T = \sum T_i,$$
$$I = \sum I_j,$$
$$F = \sum F_k.$$

Thus, refined neutrosophic attractors, repellers, and neutral sets generalize the ordinary neutrosophic versions and provide a more realistic representation of complex systems.

33. Applications

Scientific Theory Evolution
A scientific theory may be represented by

S = <T, I, F>

where:

- T = experimentally supported content,
- I = unresolved questions,
- F = falsified content.

The theory evolves dynamically as new evidence becomes available.
Social Systems
A social movement may attract some individuals, repel others, and leave many undecided.
The state naturally becomes

<attraction, neutrality, repulsion>.

Economics
Economic policies often have supporters, opponents, and undecided groups.
Neutrosophic attractors provide a direct representation of these realities.

Artificial Intelligence

Decision systems operating under contradictory information may be modeled through neutrosophic-plithogenic dynamics.

34. Conclusions and Future Research

This paper introduced Neutrosophic-Plithogenic Dynamical Systems as a generalization of classical dynamics.

The principal contribution is the extension of attractor and repeller theory from binary classifications to neutrosophic representations, and introduction of the neutrals:

$$A = <T_A, I_A, F_A>$$
$$R = <T_R, I_R, F_R>.$$

This naturally leads to:

- neutrosophic attractors,
- neutrosophic repellers,
- neutrosophic neutral sets,
- neutrosophic partial attractors,
- neutrosophic partial repellers.

The framework was further extended through neutrosophic Lyapunov exponents and neutrosophic chaos measures.

In this paper also there were introduced and investigated the Refined Attractors, Refined Neutrals, and Refined Repellers in the Neutrosophic-Plithogenic Dynamical Systems, where the components T, I, and F were split into semantic subcomponents:

$$(T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...)$$

corresponding directly to real-world attraction, neutrality, and repulsion multi-factors.

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Declarations

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Declaration of AI Use: The authors declare that the AI tool [name and version of tools] was used [for language editing and readability improvement / to assist with drafting and restructuring sections, summarizing background literature, and improving the fluency of the text / both in the writing and methodological stages / preparation, writing, or analysis] in the preparation of this manuscript. The tool did not generate original research ideas, research data, analyses, or conclusions. All content was critically reviewed, verified, and edited by the authors, who take full responsibility for the accuracy and integrity of the work. **OR** Not Applicable.
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Ethical Approval & Consent to Participate: All procedures performed in this study involving human participants followed the ethical standards of the institutional review board of [organization /university detail]. The ethics committee of the [department /institute] approved this study [reference/approval number]. All participants were informed that their responses would be kept anonymous and confidential and that the data would only be utilized for the study [study details]. **OR** Not Applicable.
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